NAG Toolbox for MATLAB

c06fb

1 Purpose

c06fb calculates the discrete Fourier transform of a Hermitian sequence of n complex data values (using a work array for extra speed).

2 Syntax

```
[x, ifail] = c06fb(x, 'n', n)
```

3 Description

Given a Hermitian sequence of n complex data values z_j (i.e., a sequence such that z_0 is real and z_{n-j} is the complex conjugate of z_j, for $j = 1, 2, \ldots, n-1$), c06fb calculates their discrete Fourier transform defined by

$$\hat{x}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} z_j \times \exp\left(-i\frac{2\pi jk}{n}\right), \qquad k = 0, 1, \ldots, n-1.$$

(Note the scale factor of $\frac{1}{\sqrt{n}}$ in this definition.) The transformed values $\hat{x}_k$ are purely real (see also the C06 Chapter Introduction).

To compute the inverse discrete Fourier transform defined by

$$\hat{y}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} z_j \times \exp\left(+i\frac{2\pi jk}{n}\right),$$

this function should be preceded by a call of c06gb to form the complex conjugates of the z_j.

c06fb uses the fast Fourier transform (FFT) algorithm (see Brigham 1974). There are some restrictions on the value of n (see Section 5).

4 References

Brigham E O 1974 *The Fast Fourier Transform* Prentice–Hall

5 Parameters

5.1 Compulsory Input Parameters

1: **x(n) – double array**

The sequence to be transformed stored in Hermitian form. If the data values z_j are written as $x_j + iy_j$, and if **x** is declared with bounds $(0 : \mathbf{n}-1)$ in the (sub)program from which c06fb is called, then for $0 \leq j \leq n/2$, x_j is contained in $\mathbf{x}(j)$, and for $1 \leq j \leq (n-1)/2$, y_j is contained in $\mathbf{x}(n-j)$. (See also Section missing entity c06background12 in the C06 Chapter Introduction and Section 9.)

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **x**.

n, the number of data values. The largest prime factor of **n** must not exceed 19, and the total number of prime factors of **n**, counting repetitions, must not exceed 20.

Constraint: $\mathbf{n} > 1$.

5.3 Input Parameters Omitted from the MATLAB Interface

work

5.4 Output Parameters

1: **x(n) – double array**

The components of the discrete Fourier transform $\hat{x}_k$. If **x** is declared with bounds $(0 : \mathbf{n} - 1)$ in the (sub)program from which c06fb is called, then $\hat{x}_k$ is stored in $\mathbf{x}(k)$, for $k = 0, 1, \ldots, n - 1$.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

At least one of the prime factors of **n** is greater than 19.

ifail $= 2$

n has more than 20 prime factors.

ifail $= 3$

On entry, $\mathbf{n} \leq 1$.

ifail $= 4$

An unexpected error has occurred in an internal call. Check all (sub)program calls and array dimensions. Seek expert help.

7 Accuracy

Some indication of accuracy can be obtained by performing a subsequent inverse transform and comparing the results with the original sequence (in exact arithmetic they would be identical).

8 Further Comments

The time taken is approximately proportional to $n \times \log n$, but also depends on the factorization of n. c06fb is faster if the only prime factors of n are 2, 3 or 5; and fastest of all if n is a power of 2.

9 Example

```
x = [0.34907;
     0.5489000000000001;
     0.74776;
     0.94459;
     1.1385;
     1.3285;
     1.5137];
```

```
[xOut, ifail] = c06fb(x)
```

```
xOut =
    1.8262
    1.8686
   -0.0175
    0.5020
   -0.5987
   -0.0314
   -2.6256
ifail =
           0
```